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Legal Gaps and their Logical Forms

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Abstract:
The concept of legal gap is tackled from a number of logical perspectives and semantic methods. After presenting our own goal (Section 1), a first introduction into legal logic refers to Bobbio's works and his formalization of legal statements (Sections 2 and 3). Then Woleński's contribution to the area is taken into account through his reference to the distinction between two juridical systems (viz. Common Law vs Civil Law) and the notion of conditional norms (Section 4). The notion of reason is also highlighted in the case of Raz's legal logic, thereby leading to a future connection with von Wright's logic of truth and an analogy made with an anti-realist reading of truth-values and norms (Section 5). Our personal contribution is introduced through a reflection on how logic should deal with the logical form of norms (Section 6), before entering a number of crucial definitions and distinctions for the concepts of norm, legal statement, and promulgation (Section 7). The final point is a proposed semantics for legal statements, which is both many-valued and gap-friendly (Section 8). A distinction between a number of requirements for permission and forbiddance leads to a set of non-classical juridical systems in which non-permission and forbiddance are not equivalent with each other any more; this does justice to Woleński's former distinction between Common Law and Civil Law, also

leading ultimately to a non-classical square of legal oppositions in which several legal operators may collapse into other ones (Section 9).
Keywords: forbiddance, juridical system, legal gap, legal statement, permission.

1. Introduction

There are two situations in a juridical system that generates century-old discussions. The first one is as follows: a child is drowning in a lake and a passerby is seeing the drama. He has to react accordingly: Ought he to save the child? Yes: there is an obligation to save someone in mortal danger, otherwise one is accused of failure to render assistance to a person in danger. No: it is forbidden to swim in this lake, according to the local *juridical norm*. Does this mean that the passerby is both obliged and forbidden to enter the lake in order to save the child? The other situation is subtly different. Like the first situation, there is a child drowning and a passerby. This time, however, there isn't any legal obligation, neither to save a person in mortal danger nor to refrain from swimming in the lake. In this case, what is to be done from the juridical logic perspective? Does anything goes, or is there some kind of constraint? The first case is a classical example of an *antinomy*, which correlates to legal *inconsistency*, while the second is an example of a *legal gap*, which corresponds to what is called legal *incompleteness*.

Our questioning is twofold: What does *legal gap* mean in a formal context? And how to deal with it from a logical perspective? The first perspective is the *positivist* approach stating that there are no *real legal gaps* but only *ideological gaps*, insofar as law is understood as a juridical system that is taken to be consistent and complete. The second perspective is the rhetorical approach stating that there exists real legal gaps ; now they are all solvable only by rhetorical means, since law is not a logical system but an *almost-logic* one. The third perspective states that gap is to *paracomplete* juridical reasoning what legal *glut* is to *paraconsistent* juridical reasoning. Our point is that there are at least three juridical answers in front of this state of affairs. Either there is always a criteria (e.g. hierarchical, chronological, specialty, competency etc.) that decides which juridical norm should be used to give only one deontic normative value for an action, i.e., the passerby is either obliged or forbidden to save the child. Or there is no such criteria, by which a conflict between two different deontic normative values arises. This conflict is one of the following two: the passerby is both obliged and forbidden or the passerby is neither obliged nor forbidden to save the child in the eyes of general law. What of this legal *pluralism*?

A many-valued system of juridical logic is proposed to account for a number of problems related to philosophy of law, especially the case of legal gaps. While a number of papers have been devoted to the case of paraconsistent legal logic (through the issues of inconsistent data bases and *defeasible reasoning*), the following wants to stress on legal gap as a case for paracomplete juridical reasoning. We propose a general framework for this purpose: AR_{4L}, which is a 4-valued juridical system including the aforementioned *juridical systems* as particular sublogics. In the vein of Von Wright's truth-logic, it consists of a formal language of juridical statements S*p*, to be read 'There is a juridical norm that states (the action described by the sentence) *p*' (where the action expressed by *p* is not indifferent in accordance with the law). Then negation may be prefixed to either S or *p*, leading to a set of 16 juridical situations on the basis of the 4 basic ones: S*p*, ¬S*p*, S¬*p*, and ¬S¬*p*. A deontic interpretation of S depends upon which kind of juridical system is mentioned with its correspondent *rule of legal closure*, whether it be Common Law ('If something is not prohibited, then it is permitted') or Civil Law ('If something is not permitted, then it is prohibited'). In the former, a *promulgation* entails that doing what a juridical norm states is permitted; whereas in the latter, a promulgation entails that doing what a juridical norm states is forbidden. Permission will be viewed as the basic deontic normative value, in the following.

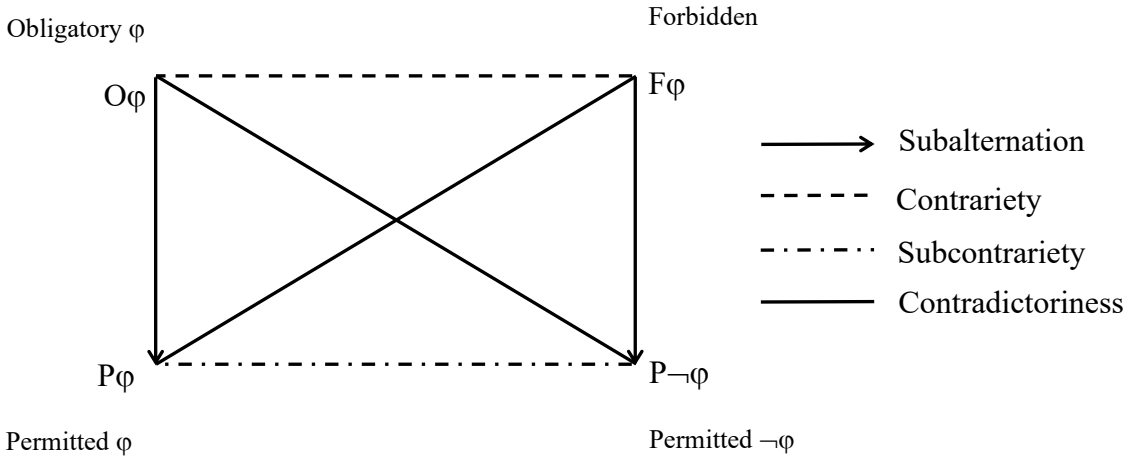
A semantics for juridical statements is an interpretation assigning deontic normative values to statements *p*, and juridical pluralism stems from the plurality of assignment conditions of them: (1) There

is a promulgation *p*, and there is a promulgation ¬*p*; (2) There is a promulgation *p*, and there is no promulgation ¬*p*; (3) There is no promulgation *p*, and there is a promulgation ¬*p*; (4) There is no promulgation *p*, and there is no promulgation ¬*p*. (1) and (4) correspond to legal gluts and gaps, and we will defend a many-valued treatment of these after a survey of the relevant literature in legal logic.

2. The Differentiation Between Antinomy and Legal Gap

An intuitive way to differentiate the problem of inconsistency from the problem of incompleteness in law is to say that the former is represented by an abundance of mutually incompatible solutions, while incompleteness represents the absence of any compatible solution [4]. In law, the problem of consistency relates to *antinomy* and the problem of completeness relates to *legal gap*.

Bobbio's analysis of antinomies involves the use of the deontic square of oppositions to demonstrate the relationships between *juridical norms*[1] in Law [1].



- **Contrariety**: two norms are contrary when it is the case that they cannot both be true, but they can both be false.
- **Contradictoriness**: two norms are contradictory when it is the case that they cannot both be true, and they cannot both be false.
- **Subcontrariety**: two norms are subcontrary when it is the case that they can both be true, but they cannot both be false.
- **Subalternation**: two norms are subalterns when it is the case that the subaltern norm (which is at the bottom of the square) always is true if the superaltern norm (which is at the top of the square) is true and if the subaltern norm is false, then the superaltern norm will be false either.

For Bobbio, two juridical norms are said to be *inconsistent* if, and only if, there are two regulations for φ and those regulations are either contraries or contradictories:

Contrary juridical norms
$O\varphi \wedge F\varphi$
Contradictory juridical norms
$O\varphi \wedge P\neg\varphi$ $F\varphi \wedge P\varphi$

What of legal gaps? In theory, a legal gap would represent a situation of *empty legal space*, that is, there would be an unregulated φ whenever there is no juridical norm that says whether φ is associated with O (obligation), F (prohibition), P (positive permission) or P¬ (negative permission). We could represent this situation as:

Legal Gap
$\neg(O\varphi \vee F\varphi \vee P\varphi \vee P\neg\varphi)$

However, there are reasons sufficiently developed by various philosophers of law to accept that empty legal space does not exist [1]. Law, as a *legal order*, has general norms with a very high degree of abstraction that can be applied to any behavior φ. We will analyze these general norms in the following section.

3. Bobbio's Insight

If there is no empty legal space, then there is only *full legal space*, and we can only think of a full legal space if the legal order has an infinity of specific norms for each possible behavior and situation or a few general norms that can be applied to a vast number of cases, if not all. Bobbio believes that the second case is the correct one to ground the legal order from a logical general theory of law.

The first general juridical norm presented by Bobbio is the *Exclusive General Norm*. This norm always accompanies a *specific norm*, much like a shadow. Consider, for example, an elevator with a sign stating that cigarette smoking, *s*, is prohibited, F*s*. What Exclusive General Norm does is dictate that, supposing that the sign constitutes the only juridical norm in place, then it is permitted to do anything else that does not involve cigarette smoking. Thus, Exclusive General Rule eliminates from the field of incidence of the specific legal norm, F*s*, any other action that is not cigarette smoking. This ensures that the legal order within the elevator example (which possesses only a single specific rule) is complete (i.e., it has no gaps) since, for every action, if the action is cigarette smoking then it is prohibited (by the specific rule), and if the action is anything other than cigarette smoking then it is permitted (by the Exclusive General Rule).

In addition to Exclusive General Norm, Bobbio identifies a second general legal norm that he calls Inclusive General Norm. The function of this norm is to ensure that actions or situations similar to those regulated by the specific norm are treated identically. To continue with our elevator example, the existence of the specific norm prohibiting cigarette smoking includes in its prohibition, acts that are similar to cigarette smoking, for example, the use of electronic cigarettes or vaporizers, hookahs or pipes, actions involving inhalation of smoke, etc. So, for every action, if the action is cigarette smoking then it is prohibited (by the specific rule), if the action is similar to cigarette smoking then it is prohibited (by Inclusive General Norm), and if the action is anything other than cigarette smoking or any action similar to it then it is permitted (by the Exclusive General Rule).

But, if the legal order is complete, once again, where are legal gaps? For Bobbio, the issue of legal gaps is not a problem of a lack of legal norms because, for every action, it will either be excluded from the scope of a specific norm by Exclusive General Norm or included into the scope of a specific norm, if it is a similar action, by Inclusive General Norm.

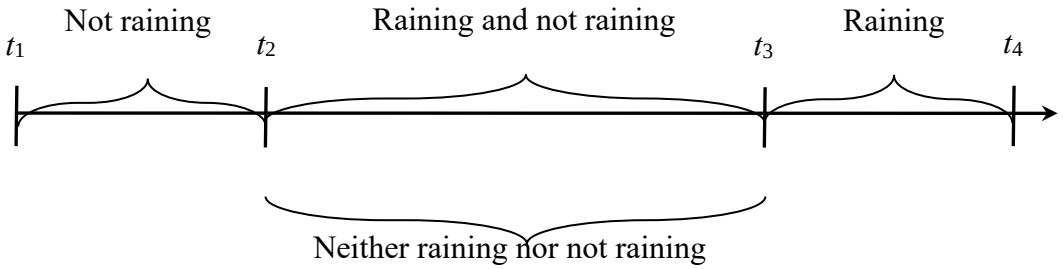
The legal gap, therefore, constitutes a problem of a more profound nature. It is evident that, in our elevator example, some situations will obviously be excluded from the scope of the cigarette smoking prohibition, like the act of whistling, while others will also quite clearly be included within the scope of

prohibition due to their similarity, like smoking a pipe. The problem arises when one encounters a borderline situation that generates quite reasonable doubts about which general norm, whether exclusive or inclusive, would be applicable.

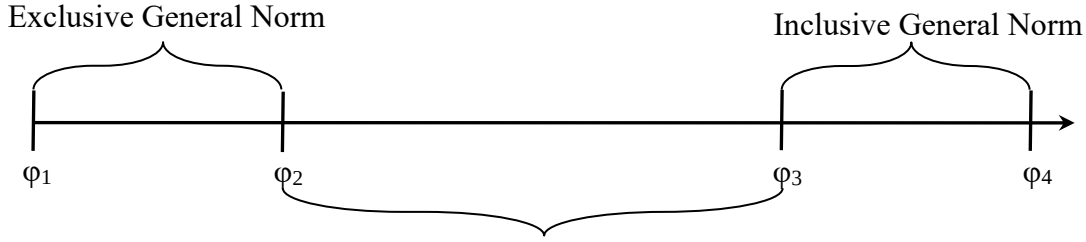
The most problematic issue related to Bobbio's reasoning is that he appears to transform the problem of legal completeness into a problem of legal consistency. It is not clear if Bobbio is aware of how much his rationale brings the legal gap and legal antinomy closer together, since the problem of completeness becomes a problem, not of lack or absence of legal norms, but of the existence of two legal norms and a doubt about which of these two legal norms should be applied. In summary, at least from the point of view of Bobbio's philosophy of law, the problem of consistency is also a problem about two legally inconsistent norms associated with a doubt about which one should be applied.

In this sense, perhaps von Wright's reference to different types of truth may help. Suppose there is a norm that provides that "If it rains, then it is prohibited to drive a car". Since rain is a natural continuous process that extends over time, there is a moment that will obviously be understood as rain, as well as another moment that will also be obviously considered as non-rain. But what about the intermediate moments, i.e. when only a few drops are falling from the sky [9]?

This question was formulated by von Wright to expose two different views of what would be a true proposition in these cases. One view understands that at this intermediate moment it is neither raining nor not raining, and another that it is both raining and not raining at the same time. Wright called the first form (*neither ... nor ...*) *strict truth*, and the second form (... and ...) *liberal truth*.



The Finnish author's thinking seems useful for understanding what Bobbio seems to mean. For Bobbio, the *intermediate zone* is the one that generates the completeness problem because, for him, this would be a situation where one cannot, from a legal point of view, choose any of the general norms for application. In this way, Bobbio seems to assume, according to von Wright's categorization, a strict view of truth. And if the judge cannot apply either of the two general rules then, indeed, in this case we would have as fact that the legal order is incomplete.



Neither Exclusive General Norm nor Inclusive General Norm

Bobbio's strict view of truth implies that, in these intermediate zone situations, a judge can ultimately resort to what is called *judiciary law*, which is a method of legal resolution that admits the judge's creative power to create the law beyond what is already established, as if the judge were the

legislator for that situation. Bobbio's view is endorsed independently by H. L. A. Hart, who asserts that legal order is always partially incomplete in possessing an open texture through which the judge, in these hard cases contained within an area of imprecision, creates the Law to be applied to the case – as if he were a legislator, albeit with greater limitations than the Legislative Power itself [2]. Bobbio understands that the application of judiciary law is a very serious matter as it violates the separation of powers, thereby generating uncertainty and disorder in democratic orders. Hart, on the other hand, views the issue in a less harmful way, understanding that, even in this creative activity, the judge acts in a much more restrained way than a real Legislative Power could act.

Bobbio's original insight is a starting point for formulating a deeper view of legal gap. However, the general norms used by Bobbio do not correspond to the closure rules most commonly worked on in current philosophy of law. In addition, his notion of Inclusive General Norm resembles the technique of using *analogy*, something admitted by the author himself, which is nothing new or revolutionary.

More problematic, however, is the fact that Bobbio's solution generates more problems than one might perceive at first reading. For instance, the application of the General Exclusive Norm on specific permissive norms seems particularly problematic, due to its ability to generate infinite prohibitions, something that will be better visualized in the example of the next section. In addition, the use of the analogy technique presents its own controversial issues, as the problem of modes of truth proposed by von Wright can affect its very use: how similar must one conduct be to another to be included in the field of incidence of the specific norm by General Inclusive Norm?

In the following section, we will seek to advance this debate and continue our deepening of legal gaps by working with the idea of closure rules.

4. Woleński's Closure Rules and the Insufficiency of Standard Deontic Logic

Like Bobbio, Woleński understands that any normative system has *closure rules*. He suggests that the two normative systems are British legalism and German legalism,[2] each of which is differentiated by its own unique closure rule, as follows [8]:

(I) if something is not prohibited then it is permitted (Common Law)
(II) if something is not permitted then it is prohibited (Civil Law)

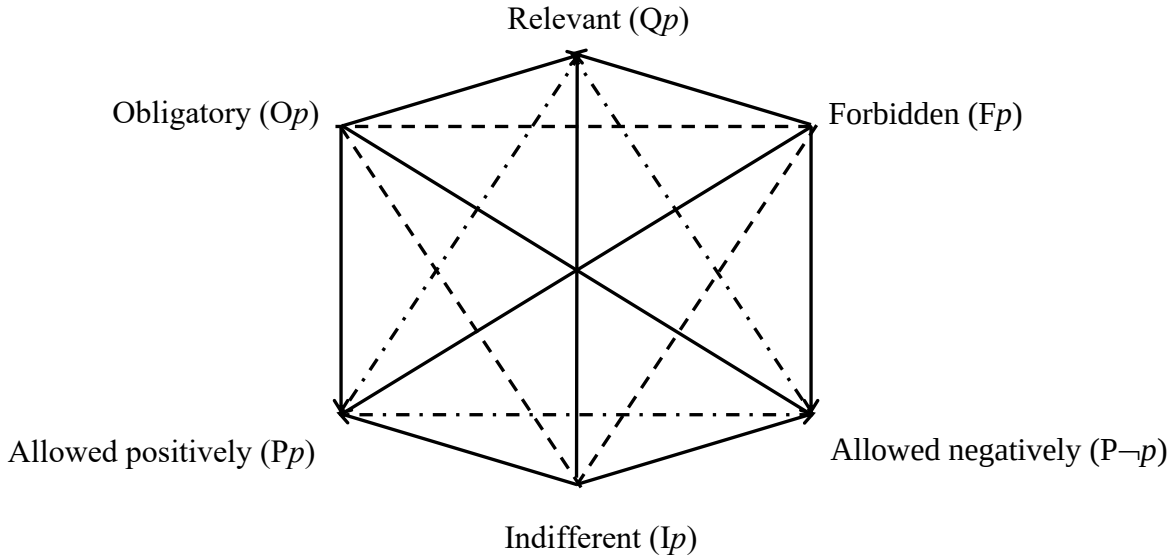
It is easy to see the similarity between these two closure rules and Bobbio's General Exclusive Norm. There may be a tendency, due to a hasty reading of Bobbio's general norms, to associate Common Law with the General Exclusive Norm, while Civil Law would be associated with General Inclusive Norm. This is a mistake, however. Closure rules (I) and (II) are actually versions of General Exclusive Norm, which excludes from the norm's field of incidence everything that is *contradictory to* what norms stipulate. Furthermore, we previously talked about the prohibition of smoking cigarettes; but consider, for example, an elevator with a sign stating that cigarette smoking, *s*, is allowed, P*s*. What Exclusive General Norm does is dictate that, supposing that the sign constitutes the only juridical norm in force, then it is prohibited to do anything else that does not involve cigarette smoking. Thus, Exclusive General Rule eliminates from the field of incidence of the permissive specific legal norm, P*s*, any other action that is not cigarette smoking. The role of Inclusive General Norm, unlike the closure rule of Common Law, is not to serve as a general permission but to authorize the use of analogy to include into the field of incidence of the specific norm, P*s*, everything that is similar to action *s*, for example, smoking pipes or cigars.

Actually, Bobbio's view contemplates only one closure rule, Exclusive General Norm, whereas Inclusive General Norm represents only the traditional method of applying analogy to fill legal gaps.

Woleński's reasoning represents an advancement, because it contemplates not one but two closure rules, both representative of the two consecrated ways of viewing the principle of legality in the history of legal reasoning. However, as we will see next, this view also has its own difficulties.

For instance, Woleński uses deontic logic to assess the applicability of (I) and (II) in legal reasoning. Deontic logic is the branch of philosophy of logic that is concerned with the study of the logical consequences of using terms like Prohibition (F), Permission (P), and Obligatory (O). There is no single language of deontic logic that is accepted unanimously, and it is a field still in frank evolution and development. However, one could say that the so-called 'Standard Deontic Logic' (SDL) is the most cited and studied system of deontic logic [7].

Suppose a proposition φ that represents some given behavior. By associating a deontic expression of prohibition with the proposition φ it is possible to infer Fφ, which is read as 'It is forbidden to φ'. The relationships between the deontic operators in SDL can be visualized in the below hexagon of oppositions [3], [8]:



In accordance with the relations of opposition, the deontic hexagon helps to derive the following:

- **Subalternation Relations**

A1. $O\varphi \rightarrow P\varphi$
A2. $F\varphi \rightarrow P\neg\varphi$
A3. $\neg P\varphi \rightarrow \neg O\varphi$
A4. $\neg P\neg\varphi \rightarrow \neg F\varphi$

- **Subcontrariety Relations**

A5. $P\varphi \wedge P\neg\varphi$
A6. $\neg(\neg P\varphi \wedge \neg P\neg\varphi)$

- **Contrariety Relations**

A7. $\neg(O\varphi \wedge F\varphi)$

- **Contradictory Relations**

A8. $F\varphi \leftrightarrow \neg P\varphi$
A9. $O\varphi \leftrightarrow \neg P\neg\varphi$

- Other important relations include:

A10. $I\varphi \rightarrow (P\varphi \vee P\neg\varphi)$

A11. $O\varphi \rightarrow Q\varphi$
A12. $F\varphi \rightarrow Q\varphi$

As can be observed, a legal system operating in accordance to a deontic logic described by this hexagon would be consistent and complete. For any φ, there would always be one unique solution.
However, the Polish philosopher noticed that for SDL (I) and (II) mean the same thing, as the below proof shows:

[1]	$\neg F\varphi \rightarrow P\varphi$	(Common Law)
[2]	$\neg P\varphi$	(Hypothesis)
[3]	$\neg P\varphi \rightarrow F\varphi$	(1, 2, MT) (Civil Law)
[4]	$(\neg F\varphi \rightarrow P\varphi) \leftrightarrow (\neg P\varphi \rightarrow F\varphi)$	

Common Law is known to validate (3), but not (1). This means that both kinds of juridical systems do not share the same definitions of the aforementioned norms. Moreover, this entails that the normal hexagon of deontic oppositions is unable to represent the separate logics of Common Law and Civil Law.

For this reason, despite its theoretical, conceptual, and pedagogical importance, the hexagon is not sufficient to translate Bobbio's intuition that there is a problem of completeness in legal systems or that there are not one, but two different closure rules. It cannot capture the legal intuition that common law systems (primacy of permission, of broad freedom) are substantially different from civil law systems (primacy of prohibition, of restricted freedom). Therefore, we will need to expand our tools beyond classical logic if we want to describe the problem of gaps in law in a formally adequate manner.

Woleński tried to understand the distinction between these two types of legalisms from the study of the specific case of competence norms, i.e., those norms that assign competences to institutions to perform acts based on what he called 'strong permissions', which would be those permissions (not generic) that are associated with specific 'conditional obligations'. The author recognized that to include this new concept he would have to extend the domains of SDL, which led him to opt for a semi-formal approach (in his own words) in order to proceed with his analysis.

In this sense, an institution would only possess those competencies that were explicitly determined (strong or explicit permissions, $P^s\varphi$) and an action of an institution that was not strongly permitted would be prohibited. Woleński defined this new primitive norm as:

$$\text{(III) } P^s\varphi \leftrightarrow (\varphi \rightarrow O^x\varphi)$$

(III) means that, if φ is done, then obligations *x* come into being. The example given by the author would be that of a committee that has permission in law to dissolve itself before the end of its members' term. In this case, this strong permission would be read as 'If the committee dissolves itself, then its decision must be respected by all, it must have the majority of votes, etc.', i.e., a permission associated with conditional obligations that result from the realization of the permitted action. However, Woleński's solution, besides being only a semi-formal solution, is also a solution that can only differentiate the common law system from the civil law system in specific cases of competence norms, which makes his approach quite restricted.

Bobbio's analysis of legal gaps is analytical, but not formal. Woleński's, on the other hand, is analytical and semi-formal. In the next section we will evaluate a formal analysis of legal gaps from an interpretation of one of the most respected philosophers of law today, Joseph Raz.

5. Raz's Legacy

In Chapter 4 of the second edition of the book *The Authority of Law*, entitled 'Legal Reasons, Sources and Gaps', Joseph Raz will express his view of what would be legal gaps, with a both analytical and formal approach [5].

One of the great problems of the analysis of gaps in versions like those of Bobbio and Woleński is that legal philosophers, when adopting the sources thesis,[3] viz. that every legal statement has a legal source that underpins it, are forced to adopt a reductive perspective of legal statements, which becomes synonymous with statements about what 'someone' orders. That is to say, if there is a legal statement, p, there is a source of law that underpins this legal statement, Sp:

$$\text{(IV)} \quad \vdash p \leftrightarrow \mathrm{S}p$$

For Raz, this entails both:

$$\text{(V)} \quad \vdash \neg p \leftrightarrow \neg \mathrm{S}p$$
$$\text{(VI)} \quad \vdash \neg p \leftrightarrow \mathrm{S}\neg p$$

And therefore, (IV) entails:

$$\text{(VII)} \quad \vdash \neg \mathrm{S}p \leftrightarrow \mathrm{S}\neg p$$

The problem is that (VII) is false, according to practical experience of law [5]. Raz's solution goes through the formalization of what he believes to be a 'conclusive reason for φ'. This statement has the form 'There is a conclusive reason for a person x to φ', and Raz uses a special logical form to represent it:

$$\text{(VIII)} \quad (\mathrm{R}_c x, \varphi)$$

Conclusive permission is formalized as:

$$\text{(IX)} \quad (\mathrm{PER}_c x, \neg\varphi)$$
$$\text{(IX}') \quad (\neg \mathrm{R}_c x, \varphi)$$

From this point, Raz philosophically assumes some conditions for this system he is developing, which are:

'There cannot be conflicting conclusive reasons':

$$\text{(X)} \quad \vdash \neg(\mathrm{R}_c x, \varphi \wedge \mathrm{R}c_x, \neg\varphi)$$

'It is not assumed that in every case to which reason applies there is a conclusive reason either for the action or against it':

$$\text{(XI)} \quad \vdash \neg(\mathrm{R}_c x, \varphi \vee \mathrm{R}_c x, \neg\varphi)$$

'A conclusive permission to act is the contradictory of a conclusive reason for refraining from that act':

$$(XII) \vdash (R_c x,\varphi) \leftrightarrow \neg(PER_c x,\neg\varphi)$$

In our understanding of what Raz proposes, there are conclusive reasons for someone to do something and there are also, let's say, partial reasons. One type of partial reason for someone to do or not to do something, and which interests us more than any other, is *legal reasons*. If the account of sources is admitted, then every legal reason has a source, '*so*'. In this way, the legal statement '*so* is a legal reason for *x* to φ' can be written

$$(XIII)\ so\mathrm{LR}x,\varphi$$

And concerning explicit permissions that, for Raz, in the same way as Woleński, constitutes a permission to cancel an existing legal reason or to forestall possible reasons by cancelling them in advance:

$$(XIV)\ so\mathrm{LPER}_c x,\neg\varphi$$

It is this type of legal statement that, for Raz, allows a deep understanding of the source thesis. The source thesis implies a truth criterion for evaluating the values of legal statements. Statements of the form *so*LR*x*,φ are true if, and only if, *so* can be substituted by a source of law (a social fact), without recourse to moral arguments. Statements of the form *so*LPER$_c$*x*,¬φ are true if, and only if, *so* can be substituted by a source of law (a social fact) that cancels a legal reason, without recourse to moral arguments. In this sense, it follows that negative legal statements (¬*so*LR*x*,φ) do not have sources, while explicit permissions (*so*LPER$_c$*x*,¬φ) always have sources.

And what does it mean to say that a legal statement does not have a source? For Raz, it means that *so* cannot be substituted by a source in a "complete" manner and, therefore, the answer to the question "What is the source (social fact) that legally substantiates this legal statement?" is non-existent or incomplete. In this way, Raz defines *legal completeness* and *legal gap* as: "A legal system is legally complete if there is a complete answer to all the legal questions over which the courts have jurisdiction. It contains a legal gap if some legal questions subject to jurisdiction have no complete answer" [5].

In other words, when the question 'What decision does the law require in this case?' is met with the answer 'No decision is required by law', then there is a legal gap. There are only two possible legal complete answers.

The law conclusively requires that action:

$$(XV)\ \mathrm{LR}_c x,\varphi$$

The law conclusively permits the omission of that action:

$$(XVI)\ \mathrm{LPER}_c x,\neg\varphi$$

Conclusive permission can be written as follows:

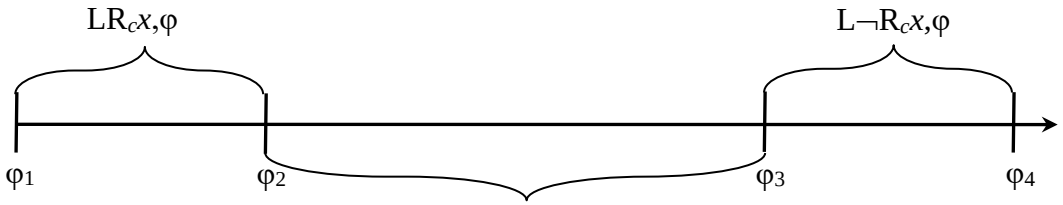
$$(XVI')\ \mathrm{L}\neg\mathrm{R}_c x,\varphi$$

Therefore, if there are two types of conclusive legal responses then there are two types of gap:

$$(XVII)\ \mathrm{LR}_c x,\varphi \text{ is neither true or false and } \mathrm{L}\neg\mathrm{R}_c x,\varphi \text{ is neither true or false}$$

$$(XVIII)\ \neg\mathrm{LR}_c x,\varphi \wedge \neg\mathrm{L}\neg\mathrm{R}_c x,\varphi \text{ is true}$$

Note that (XVII) exactly reflects Bobbio's insight from a strict perspective of truth, now formalized:



The gap (XVII) is not exactly a result of a problem with legal language but, rather, with the nature of natural language as a whole. It shows that, in borderline cases, the interpreter of the law simply does not have a legal criterion to determine whether to include conduct in the scope of a conclusive norm that requires φ or a permissive norm that allows ¬φ.

This is particularly problematic in cases where the laws themselves use adjectives or moral terminology to define actions [5]. For example, saying that a murder committed with cruelty will have a corresponding increased sentence that will certainly bring the judge to the impossibility of deciding by exclusively legal means. In these cases, it is likely that case law, or the judge's discretion, will determine the solution (of the definition of cruelty), with a low degree of legal certainty.

Raz's advancement over Bobbio is that his formalization allows for a clearer distinction between antinomy and legal gaps. While (XVII) and (XVIII) represent types of legal gaps, the antinomy would be represented as [5]:

$$\text{(XIX) } \mathrm{LR}_c x,\varphi \wedge \mathrm{LR}_c x,\neg\varphi$$

According to Raz, the legal gap presented by (XVIII) would be solved with a closure rule, more precisely, Raz's version of Common Law legalism:

$$\text{(XX) } \neg\mathrm{LR}_c x,\varphi \rightarrow \mathrm{L}\neg\mathrm{R}_c x,\varphi$$
$$\text{(XX}') \ \neg\mathrm{LR}_c x,\varphi \rightarrow \mathrm{LPER}_c x,\neg\varphi$$

If (XX) is used to replace ¬LR$_c$x,φ in (XVIII), we get:

$$\text{(XXI) } \mathrm{L}\neg\mathrm{R}_c x,\varphi \wedge \neg\mathrm{L}\neg\mathrm{R}_c x, \varphi \text{ is true}$$
$$\text{(XXI}') \ \mathrm{LPER}_c x,\neg\varphi \wedge \neg\mathrm{LPER}_c x,\neg\varphi \text{ is true}$$

(XXI) and (XXI′) are contradictions and, because of that, Raz rejects them. Note that these expressions do not represent antinomies, since the problem is not the conflict between contrary or contradictory legal reasons equally valid ; rather, the conflict falls on the existence or not of legal reasons grounded by sources of law.

However, Raz's solution is also unable to resolve the distinction between the legalisms of Common Law and Civil Law. If we used Raz's formalism to describe (I) and (II), we would get:

[1]	$\neg\mathrm{LR}_c x,\varphi \rightarrow \mathrm{L}\neg\mathrm{R}_c x,\varphi$	[(XX)]
[2]	$\neg\mathrm{LR}_c x,\varphi \rightarrow \mathrm{LPER}_c x,\neg\varphi$	[(XX′)], (Civil Law)
[3]	$\neg\mathrm{LPER}_c x,\neg\varphi \rightarrow \mathrm{LR}_c x,\varphi$	[2, MT], (Common Law)

[2] could be read as 'That which is not legally conclusively required is legally conclusively permitted not to do', while [3] could be read as 'That which is not legally conclusively permitted not to do is conclusively required to do'. Thus, in Raz's system [2] is equivalent to (I) and [3] is equivalent to (II), and both [2] and [3] are equivalent in Raz's formalization, as [I] and [II] are equivalent in SDL.
For this reason, in the following sections, we will seek to construct a legal logic that is capable of making sense of the two closure rules warranting, in theory, the completeness of legal systems. If it is not clear by now, we will discuss, from now on, just the legal gap in the sense expressed by (XVIII).

6. Logic and Norms

Let φ be an arbitrary sentence. It is taken for granted that not every kind of sentence is entitled to be a legal statement, that is, an information whose content is relevant with respect to the social facts that ground the law (sources). If φ stands for the sentence 'I am watching TV', it seems clear that φ is not entitled to become a legal statement; whereas, if φ is 'My neighbour killed his dog', then the sentential content of φ is relevant and should be of concern for the law. It is also taken for granted that killing an animal is either permitted or forbidden, given the special circumstance at which the fact occurred (the neighbour may have killed in the name of legitimate self-defence because his dog contracted rabies, for example).

Whilst it is not the job of logic to discriminate between ordinary sentences and legal statements, i.e., those sentences that are juridically relevant or are not, the aim of a proper legal logic is to explain what follows from such legal statements.

7. Legal Logic

In order to know whether φ is a legal statement or not, it suffices to see whether φ is made explicit or *stated* or not by an arbitrary source, that is, any source that states φ and thereby makes φ a legal statement. In other words:

Definition 1. A sentence φ is a *legal statement* if, and only if, there is a source (henceforth: S) that states φ as a norm.

Let us call this a *promulgation*, i.e. whenever a source states a sentence in its set of explicit norms. In symbols:

$$S\varphi =_{df} \exists S\ \varphi \in S$$

This also means that a sentence is not a legal statement whenever no source states it. It is said indifferent [8], such that:

Definition 2. A sentence φ is said *indifferent* if, and only if, there is no source that states φ as a norm.

This means that an indifferent sentence is whatever is not a legal statement. In symbols:

$$I\varphi =_{df} \neg\exists S\ \varphi \in S$$

However, a difference is to be made between two ways of denying the legal status of an arbitrary sentence φ. It may be so because φ does not belong to the set of relevant data that have to do with the sources, as

e.g. “I am watching TV”. If so, then it is the case that Iφ. It may also be because the sentential content is condemned by a given source, so the sentential content is promulgated by that source. If so, then we have a situation where S¬φ holds. The logical relation between legal statements is such that, for any given source S embedding φ, three main properties may be established about the behavior of logical negation.

Definition 3. For any legal statement Sφ, the negative promulgation of a negative sentence amounts to promulgating an affirmative sentence. In symbols:

$$S\neg(\neg\varphi) \leftrightarrow S\varphi$$

The lack of not promulgating a legal statement amounts to promulgating it. In symbols:

$$\neg(\neg S\varphi) \leftrightarrow S\varphi$$

Promulgating a negative sentence φ entails not promulgating the corresponding affirmative sentence φ. In symbols:

$$S\neg\varphi \rightarrow \neg S\varphi$$

A set of norms can then be defined accordingly, once legal statements are identified by means of the above definition. These norms are: obligation, O, forbiddance, F, and permission, P. An usual logical analysis of these is exemplified by the square of deontic oppositions, as presented by e.g. [8]. Thus, for any legal statement φ:

$O\varphi \leftrightarrow \neg P\neg\varphi$
$F\varphi \leftrightarrow O\neg\varphi$

The normal square of deontic oppositions lists a set of ensuing logical relations between legal statements, accordingly:

(i) $O\varphi \rightarrow P\varphi$
(ii) $O\varphi \rightarrow \neg F\varphi$
(iii) $\neg O\varphi \rightarrow P\neg\varphi$
(iv) $F\varphi \rightarrow P\neg\varphi$
(v) $F\varphi \rightarrow \neg O\varphi$
(vi) $\neg F\varphi \rightarrow P\varphi$
(vii) $P\varphi \rightarrow \neg F\varphi$
(viii) $\neg P\varphi \rightarrow F\varphi$

Now there is a distinction between two kinds of legal systems, namely: *Common Law* and *Civil Law*, that do not validate any of the above theorems (i)-(viii). Common Law is known to validate (vi), but not (8); and Civil Law is known to validate (viii), but not (vi). This means that both kinds of law do not share the same definitions of the aforementioned norms. Moreover, this entails, as demonstrated throughout the present paper, that the normal square (or hexagon) of deontic oppositions is unable to represent the logic of Common Law and Civil Law: the normal square stands for a ‘classical’ set of norms that are both complete and consistent with respect to one unique given juridical system, S.

We still need to define what a juridical system is, in order to account for Common Law, Civil Law, and non-normal situations such as legal gap. This can be account by the fact that a system is a set of several distinctive laws, and that the latter may be at odds about the range of legal statements.

Definition 3. A *juridical system* S is a finite set of sources $S_1,...,S_n$ establishing logical conditions for assigning the norms P and F to a legal statement φ.

The set of logical relations (i)-(viii) depicts a special kind of juridical system, namely: legally consistent and complete. The logical properties of legal consistency and completeness can be defined in terms of basic legal statements. Thus:

Definition 4. A juridical system S is said S-*consistent* only if, for any legal statement φ, φ cannot be both stated and not stated by any source. In symbols:

$$\vDash_{\mathsf{S}} \varphi \text{ only if } \nvDash_{\mathsf{S}} \neg\varphi$$

Definition 5. A juridical system S is said S-*complete* only if, for any legal statement φ, φ cannot be neither stated nor not stated by any source. In symbols:

$$\nvDash_{\mathsf{S}} \varphi \text{ only if } \vDash_{\mathsf{S}} \neg\varphi$$

The following wants to show that the complete and consistent juridical system is just one among other ones. The way to construct and compare such juridical systems is the aim of the following non-classical juridical logic.

8. Non-Classical Juridical Logic

We are especially interested in a special case of normative drawback: legal *gap*. A legal gap is a situation in which the juridical system is not S-complete, that is:

$$\nvDash_{\mathsf{S}} \varphi \text{ and } \nvDash_{\mathsf{S}} \neg\varphi$$

Whilst such a gappy situation is made impossible by S-consistent and S-complete juridical systems, it can make sense in juridical systems that infringe any of these "classical" properties.

8.1. Common Civil and Civil Law

The central problem is about how to define one and the same norm, i.e., permission. In Common Law, any legal statement that is not forbidden is thereby permitted. But that is not the case in Civil Law, whereby a legal statement is permitted only if the source has promulgated that it is not forbidden. In order to show that asymmetry between both juridical systems, we want to afford a list of distinct definitions of juridical norms that parallel other works about the logic of truth [9] and epistemic criteria of justification [6].

Here is a set of four juridical systems, S_1-S_4, each being a specific way of dealing logically with a set of sources S. Thus, for any legal statement φ, the conditions of permission and forbiddance for φ in a given juridical system differ as follows:

Definition 7. The juridical system S_1 is such that, for any legal statement φ that belongs to S_1:

- φ is permitted in S_1 if, and only if, either its sentential content *p* is promulgated or its negation ¬*p* is not promulgated. In symbols:
  $\vDash_{S_1} P\varphi$ iff $\vDash_{S_1} Sp$ or $\vDash_{S_1} \neg S\neg p$
- φ is forbidden in S_1 if, and only, both its sentential content *p* is not promulgated and its negation ¬*p* is promulgated. In symbols:
  $\vDash_{S_1} F\varphi$ iff $\vDash_{S_1} S\neg p$ and $\vDash_{S_1} \neg Sp$

Definition 8. The juridical system S_2 is such that, for any legal statement φ that belongs to S_2:

- φ is permitted in S_2 if, and only, both its sentential content *p* is promulgated and its negation ¬*p* is not promulgated. In symbols: $\vDash_{S_2} P\varphi$ iff $\vDash_{S_2} Sp$ and $\vDash_{S_2} \neg S\neg p$
- φ is forbidden in S_2 if, and only, either its sentential content *p* is not promulgated or its negation ¬*p* is promulgated. In symbols: $\vDash_{S_2} F\varphi$ iff $\vDash_{S_2} S\neg p$ or $\vDash_{S_2} \neg Sp$

Definition 9. The juridical system S_3 is such that, for any legal statement φ that belongs to S_3:

- φ is permitted in S_3 if, and only, both its sentential content *p* is promulgated and its negation ¬*p* is not promulgated. In symbols: $\vDash_{S_3} P\varphi$ iff $\vDash_{S_3} Sp$ and $\vDash_{S_3} \neg S\neg p$
- φ is forbidden in S_3 if, and only, both its sentential content *p* is not promulgated and its negation ¬*p* is promulgated. In symbols: $\vDash_{S_3} F\varphi$ iff $\vDash_{S_3} S\neg p$ or $\vDash_{S_3} \neg Sp$

Definition 10. The juridical system S_4 is such that, for any legal statement φ that belongs to S_4:

- φ is permitted in S_4 if, and only, either its sentential content *p* is promulgated or its negation ¬*p* is not promulgated. In symbols: $\vDash_{S_4} P\varphi$ iff $\vDash_{S_4} Sp$ or $\vDash_{S_4} \neg S\neg p$
- φ is forbidden in S_4 if, and only, either its sentential content *p* is not promulgated or its negation ¬*p* is promulgated. In symbols: $\vDash_{S_4} F\varphi$ iff $\vDash_{S_4} S\neg p$ or $\vDash_{S_4} \neg Sp$

Now it can be easily proved that none of the juridical systems S_1-S_4 validates only one of the characteristic formulas of Common Law or Civil Law, however: both are valid in S_1, S_2 and S_4, whereas none is valid in S_3. The latter is an incomplete or 'gappy' juridical system, in accordance with our expected case of legal gap. But, we still want juridical systems providing with different criteria for permission and forbiddance. A characteristic semantics for Common Civil and Civil Law is still in order, accordingly.

8.2. Semantics for Common Law and Civil Law

Definition 11. Each legal statement φ is interpreted as an ordered pair of states about whether φ or its negation ¬φ is promulgated, such that its characteristic value is of the form:

$$v(\varphi) = \langle S\varphi, S\neg\varphi \rangle$$

Definition 12. Juridical bivalence: Any legal statement or its negation is to be promulgated, and no legal statement can be promulgated and not be promulgated at once. That is, for any φ:

Either Sφ or S¬φ
Not Sφ and S¬φ

φ	¬φ

11	11
10	01
01	10
00	00

Juridical system of Common Law: S_{CoL}

- φ is permitted in S_{CoL} if, and only if, either its sentential content φ is promulgated or its negation ¬φ is not promulgated. In symbols:
  $\vDash_{S_{CoL}} P\varphi$ iff $\vDash_{S_{CoL}} S\varphi$ or $\vDash_{S_{CoL}} \neg S\neg\varphi$
- φ is not permitted in S_{CoL} if, and only if, either its sentential content φ is not promulgated or its negation ¬φ is promulgated. In symbols:
  $\vDash_{S_{CoL}} \neg P\varphi$ iff $\vDash_{S_{CoL}} \neg S\varphi$ or $\vDash_{S_{CoL}} S\neg\varphi$
- φ is forbidden in S_{CoL} if, and only if, its sentential content φ is not promulgated and its negation ¬φ is promulgated. In symbols:
  $\vDash_{S_{CoL}} F\varphi$ iff $\vDash_{S_{CoL}} \neg S\varphi$ and $\vDash_{S_{CoL}} S\neg\varphi$
- φ is not forbidden in S_{CoL} if, and only if, its sentential content φ is promulgated and its negation ¬φ is not promulgated. In symbols:
  $\vDash_{S_{CoL}} \neg F\varphi$ iff $\vDash_{S_{CoL}} S\varphi$ and $\vDash_{S_{CoL}} \neg S\neg\varphi$

φ	Pφ	¬Pφ	Fφ	¬Fφ
11	1	1	0	0
10	1	0	0	1
01	0	1	1	0
00	1	1	0	0

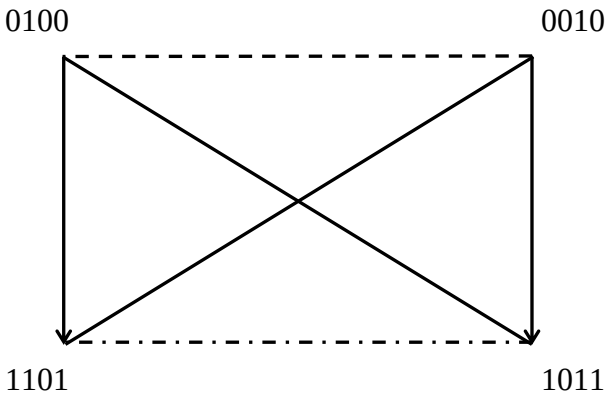
Juridical system of Civil Law: S_{CiL}

- φ is permitted in S_{CiL} if, and only if, either its sentential content φ is promulgated and its negation ¬φ is not promulgated. In symbols:
  $\vDash_{S_{CiL}} P\varphi$ iff $\vDash_{S_{CiL}} S\varphi$ or $\vDash_{S_{CiL}} \neg S\neg\varphi$
- φ is not permitted in S_{CiL} if, and only if, either its sentential content φ is not promulgated and its negation ¬φ is promulgated. In symbols:
  $\vDash_{S_{CiL}} \neg P\varphi$ iff $\vDash_{S_{CiL}} \neg S\varphi$ or $\vDash_{S_{CiL}} S\neg\varphi$
- φ is forbidden in S_{CiL} if, and only if, its sentential content φ is not promulgated or its negation ¬φ is promulgated. In symbols:
  $\vDash_{S_{CiL}} F\varphi$ iff $\vDash_{S_{CiL}} \neg S\varphi$ and $\vDash_{S_{CiL}} S\neg\varphi$
- φ is not forbidden in S_{CiL} if, and only if, its sentential content φ is promulgated or its negation ¬φ is not promulgated. In symbols:
  $\vDash_{S_{CiL}} \neg F\varphi$ iff $\vDash_{S_{CiL}} S\varphi$ and $\vDash_{S_{CiL}} \neg S\neg\varphi$

φ	Pφ	¬Pφ	Fφ	¬Fφ
11	0	0	1	1
10	1	0	0	1
01	0	1	1	0
00	0	0	1	1

9. A Non-Classical Square of Legal Oppositions

From the aforementioned study, we can represent the relationships of S_{CoL} and S_{CiL} in a non-classical square of legal oppositions that includes the legal statements and their logical relationships.



Description of the nodes in the non-classical square of legal oppositions

0100: $PS_{CiL}\varphi$, $\neg PS_{CiL}\neg\varphi$, $FS_{CoL}\neg\varphi$, $\neg FS_{CoL}\varphi$
1101: $PS_{CoL}\varphi$, $\neg PS_{CoL}\neg\varphi$, $FS_{CiL}\neg\varphi$, $\neg FS_{CiL}\varphi$
0010: $PS_{CiL}\neg\varphi$, $\neg PS_{CiL}\varphi$, $FS_{CoL}\varphi$, $\neg FS_{CoL}\neg\varphi$
1011: $PS_{CoL}\neg\varphi$, $\neg PS_{CoL}\varphi$, $FS_{CiL}\varphi$, $\neg FS_{CiL}\neg\varphi$

The 'non-classical' import of the above square is obviously due to the previous definitions of permission and forbiddance. More precisely, it is related to the non-dual relationship between the corresponding operators P and F. It has been recalled in the following tables that forbiddance does not amount to the lack of permission in Common Law and Civil Law: non-permission implies forbiddance in the latter, whereas non-forbiddance implies permission in the former; but the converse never holds in both juridical systems, unlike the square (and its hexagonal extension) of deontic oppositions wherein norms behave like normal modal operators.

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Notes

1. The author does not use the term “normative proposition”, but the term “norm”, not discussing the distinction between them, i.e., the descriptive and prescriptive senses of norms [10].
2. From this point onward, we will call British legalism ‘Common Law‘ and German legalism ‘Civil Law.’
3. For Raz, sources are basically the origin of the existence and content of Law, and any legitimate legal decision refers to these objective social facts (the sources) and not to moral considerations.